

NUMERICAL SOLUTIONS OF THE SYMMETRIC REGULARIZED-LONG-WAVE EQUATION BY TRIGONOMETRIC INTEGRATOR PSEUDOSPECTRAL DISCRETIZATION

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ABSTRACT. The computation of the symmetric regularized-long-wave (SRLW) equation, which describes weakly nonlinear ion acoustic and space-charge waves, is dealt with in this paper. The numerical scheme to be proposed applies the Fourier pseudospectral discretization to spatial derivatives in time space, with time advance accomplished in phase space by a integrator based on trigonometric polynomials which is fully explicit. Extensive numerical tests are reported, which are geared towards understanding the accuracy properties and studying the collisions of solitary waves in the SRLW. The results suggest that the scheme is of spectral-order accuracy in space and second-order accuracy in time. Also, some intriguing phenomena in the solitary wave collisions are observed in simulations.

1. INTRODUCTION

Consider the symmetric regularized-long-wave (SRLW) equation,

$$(1.1) \quad u_{tt} - u_{xx} + \frac{1}{2} (u^2)_{xt} - u_{xxtt} = 0, \quad x \in \mathbb{R}, \quad t > 0,$$

which was first derived by Seyler and Fenstermacher [13] from a weakly nonlinear analysis of the cold-electron plasma equations appropriate for a strongly magnetized nonrelativistic electron beam such that the fluid motion is constrained to one direction. This equation was shown to describe weakly nonlinear ion acoustic and space-charge waves, and the real-valued $u(x, t)$ corresponds to the dimensionless fluid velocity with a decay condition $\lim_{|x| \rightarrow \infty} u = 0$.

The SRLW equation is explicitly symmetry in the x and t derivatives, and is very similar to the regularized-long-wave equation which describes shallow water waves and plasma drift waves [1, 2]. The existence and uniqueness of solutions for the SRLW have been established, e.g., in [4, 5]. It is well-known that the SRLW equation (1.1) possesses the solitary waves of the form [5, 13]

$$(1.2) \quad u_c(x, t) = \frac{3(c^2 - 1)}{c} \operatorname{sech}^2 \left(\sqrt{\frac{c^2 - 1}{4c^2}} (x - ct) \right).$$

Note that the two branches of solitary waves for the velocity c in the ranges $c > 1$ and $c < -1$ corresponds to the right and the left traveling solitary waves of the same type, and hence the SRLW equation describes bidirectional propagation.

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The SRLW equation (1.1) has an equivalent form as a first-order system:

$$(1.3) \quad u_t + \rho_x + \frac{1}{2}(u^2)_x - u_{xxt} = 0,$$

$$(1.4) \quad \rho_t + u_x = 0, \quad x \in \mathbb{R}, \quad t > 0,$$

from which one can determine four invariants of the motion for the SRLW [5, 13]:

$$(1.5) \quad I(u) = \int_{-\infty}^{\infty} u(x, t) dx, \quad I(\rho) = \int_{-\infty}^{\infty} \rho(x, t) dx,$$

$$(1.6) \quad E(u, \rho) = \int_{-\infty}^{\infty} (u^2(x, t) + u_x^2(x, t) + \rho^2(x, t)) dx,$$

$$(1.7) \quad V(u, \rho) = \int_{-\infty}^{\infty} \left(u(x, t)\rho(x, t) + \frac{1}{6}u^3(x, t) \right) dx.$$

As pointed out in [13], the system (1.3)-(1.4) does not possess the Painlevé property which is a sufficient condition for integrability and (1.3)-(1.4) has only a finite number of polynomial conservation laws, all of which are represented by (1.5)-(1.7).

In this paper, we are going to address the numerics of the SRLW. Due to its wave-type shape and the decay condition, the spectral-type approach has been extensively used for the SRLW. For instance, in [4, 16] the Fourier spectral/pseudospectral discretization was applied to solving the equivalent first-order system (1.3)-(1.4) with the time advance accomplished by backwards Euler method. Another category of the numerical methods available in literatures for the SRLW is the time-domain finite difference (TDFD) methods. Among of them three conservative schemes were proposed and analyzed in [15], which conserve some discrete quantities analogous to (1.5) and (1.6). The key point in designing conservative TDFD schemes for wave-type differential equations is to use an appropriate temporal discretization, like the classical Crank-Nicolson method, and the rigorous proof of conservation proceeds by means of ‘discrete energy method’ with the help of summation by parts formula.

We would like to comment that one can readily obtain the spectral-type conservative schemes for the SRLW by combining the Fourier spectral/pseudospectral discretization with the finite difference temporal discretization used in conservative TDFD methods. The conservative quantities can be proven by analogous arguments as TDFD noting that the integration by parts holds in the finite-dimensional functional space spanned by Fourier bases and summation by parts also holds in the vector space spanned by discrete Fourier bases [7, 9, 12, 14]. Although conservative schemes have nice mathematical properties, in practice at each step certain fully nonlinear problem has to be solved very accurately which requires many iterations in implementation and is quite time-consuming. If the nonlinear problem is not solved very accurately, the conservation does not hold in computation (see e.g., [3]). In the view of these observations, the spectral-type conservative scheme for the SRLW is not examined in this paper.

The aim of this work is to propose an efficient and accurate numerical discretization for the SRLW equation (1.1), and apply the scheme to study the collisions of solitary waves in various setups. The scheme to be proposed is based on the application of Fourier pseudospectral discretization to spatial derivatives, followed by a numerical integration via trigonometric polynomials to the resulting second-order nonlinear ODEs for the Fourier coefficients in phase space. The numerical integration first writes out the exact solutions of the second-order nonlinear ODEs in integral form, and then approximates the integrals by proper numerical quadratures such that the resulting scheme is fully explicit. Such integrator is

somewhat similar to a Gautschi-type exponential integrator [8, 10, 11] and a Deuffhard-type integrator [6, 11], which have been successfully used in the semi-discretization of wave-type equations (see, e.g., [3, 10]) and approximates much better than frequently used finite difference integrators under the same time step (see, e.g., [3]). But it also differs from the standard Gautschi-type or Deuffhard-type integrator, since the integrands arising in the mixed derivative term in (1.1) suggest a four-level scheme while the standard Gautschi-type or Deuffhard-type integrator for wave-type equations results in a three-level scheme.

The rest is organized as follows. In Section 2 the efficient numerical method is proposed in very details. In Section 3 we first test the accuracy properties of the proposed method, and then apply it to study the collisions of solitary waves in the SRLW. Finally, some concluding remarks are drawn in Section 4.

2. NUMERICAL METHOD

In this section we shall propose a numerical discretization for the SRLW (1.1). In practice, we truncate the whole space problem (1.1) into an interval $\Omega = (a, b)$ with periodic boundary conditions, i.e., consider the following initial-boundary-value problem:

$$(2.1) \quad u_{tt} - u_{xx} + \frac{1}{2}(u^2)_{xt} - u_{xxtt} = 0, \quad x \in \Omega, \quad t \geq 0,$$

$$(2.2) \quad u(x, 0) = u^{(0)}(x), \quad u_t(x, 0) = u^{(1)}(x), \quad x \in \bar{\Omega} := [a, b],$$

$$(2.3) \quad u(a, t) = u(b, t), \quad u_x(a, t) = u_x(b, t), \quad t \geq 0.$$

Such boundary conditions are inspired by the decay condition $\lim_{|x| \rightarrow \infty} u = 0$. Choose mesh size $h := (b - a)/M$ with M an even positive integer, time step $\tau > 0$, and denote the mesh points with coordinates $(x_j, t_n) = (a + jh, n\tau)$ for $j = 0, 1, \dots, M$ and $n = 0, 1, \dots$. In what follows we will approximate the spatial derivatives in time space along the Fourier pseudospectral approximation path, followed by advancing the time in phase space by a numerical integration based on trigonometric polynomials.

Assume

$$(2.4) \quad u(x, t) \approx \sum_{l=-M/2}^{M/2-1} \tilde{u}_l(t) \exp(i\mu_l(x - a)),$$

for $x \in \bar{\Omega}$ and $t \geq 0$, with $\tilde{u}_l(t)$ the discrete Fourier transform coefficient of the l th mode.

$$\tilde{u}_l(t) = \frac{1}{M} \sum_{j=0}^{M-1} u(x_j, t) \exp\left(-\frac{2ijl\pi}{M}\right), \quad \mu_l = \frac{2\pi l}{b - a}, \quad l = -M/2, \dots, M/2 - 1.$$

Approximating the spatial derivatives in (2.1) by Fourier pseudospectral discretization (cf. [7, 9, 12, 14]), i.e.,

$$-\partial_{xx}u(x, t) \approx \sum_{l=-M/2}^{M/2-1} \mu_l^2 \tilde{u}_l(t) \exp(i\mu_l(x - a)), \quad \partial_x u(x, t) \approx \sum_{l=-M/2}^{M/2-1} i\mu_l \tilde{u}_l(t) \exp(i\mu_l(x - a)),$$

and noting orthogonality of the Fourier functions, we obtain the following second-order ODEs in phase space, for $l = -M/2, \dots, M/2 - 1$,

$$(2.5) \quad \frac{d^2}{dt^2} \tilde{u}_l(t) + \mu_l^2 \tilde{u}_l(t) + \frac{i\mu}{2} \frac{d}{dt} (\widetilde{u^2})_l(t) + \mu_l^2 \frac{d^2}{dt^2} \tilde{u}_l(t) = 0.$$

Now we discuss a numerical integration for (2.5) in very details.

2.1. Integration based on trigonometric polynomials. For t_n ($n = 0, 1, \dots$) any given time and $s \in \mathbb{R}$, we rewrite the ODEs (2.5) as

$$\frac{d^2}{ds^2} \tilde{u}_l(t_n + s) + \beta_l^2 \tilde{u}_l(t_n + s) + \frac{i\mu_l}{2(1 + \mu_l^2)} \tilde{f}_l^n(s) = 0,$$

where,

$$\beta_l = \frac{|\mu_l|}{\sqrt{1 + \mu_l^2}}, \quad \tilde{f}_l^n(s) = \widetilde{(u^2)_l}(t_n + s).$$

The analytical solutions of the above second-order ODEs can be written explicitly thanks to variation-of-constants formula,

$$(2.6) \quad \tilde{u}_l(t_n + s) = \cos(\beta_l s) \tilde{u}_l(t_n) + \frac{\sin(\beta_l s)}{\beta_l} \tilde{u}_l'(t_n) - \frac{i\mu_l}{2\beta_l(1 + \mu_l^2)} \int_0^s \frac{d}{dw} \tilde{f}_l^n(w) \sin(\beta_l(s - w)) dw.$$

Here and after we adopt the convention $\sin(x)/x = 1$ when $x = 0$.

When $n = 0$, from the initial condition (2.2), i.e.,

$$\tilde{u}_l(0) = \widetilde{(u^{(0)})_l}, \quad \tilde{u}_l'(0) = \widetilde{(u^{(1)})_l},$$

and approximating the integral via the standard trapezoidal rule, we have

$$(2.7) \approx \tilde{u}_l(t_1) = \cos(\beta_l \tau) \widetilde{(u^{(0)})_l} + \frac{\sin(\beta_l \tau)}{\beta_l} \widetilde{(u^{(1)})_l} - \frac{i\mu_l}{2\beta_l(1 + \mu_l^2)} \int_0^\tau \frac{d}{dw} \tilde{f}_l^0(w) \sin(\beta_l(\tau - w)) dw$$

$$\approx \cos(\beta_l \tau) \widetilde{(u^{(0)})_l} + \frac{\sin(\beta_l \tau)}{\beta_l} \widetilde{(u^{(1)})_l} - \frac{i\mu_l \tau \sin(\beta_l \tau)}{2\beta_l(1 + \mu_l^2)} \widetilde{(u^{(0)} u^{(1)})_l}.$$

For $n \geq 1$, by the integration by parts,

$$\int_0^s \frac{d}{dw} \tilde{f}_l^n(w) \sin(\beta_l(s - w)) dw = -\sin(\beta_l s) \tilde{f}_l^n(0) + \beta_l \int_0^s \tilde{f}_l^n(w) \cos(\beta_l(s - w)) dw,$$

thus,

$$(2.8) \quad \begin{aligned} \tilde{u}_l(t_n + s) &= \cos(\beta_l s) \tilde{u}_l(t_n) + \frac{\sin(\beta_l s)}{\beta_l} \tilde{u}_l'(t_n) \\ &\quad - \frac{i\mu_l}{2(1 + \mu_l^2)} \left[\int_0^s \tilde{f}_l^n(w) \cos(\beta_l(s - w)) dw - \frac{\sin(\beta_l s)}{\beta_l} \tilde{f}_l^n(0) \right]. \end{aligned}$$

Adding (2.8) with $s = \tau$ to its evaluation with $s = -\tau$, we have the following three-term recurrence

$$(2.9) \quad \tilde{u}_l(t_{n+1}) + \tilde{u}_l(t_{n-1}) = 2\cos(\beta_l \tau) \tilde{u}_l(t_n) - \frac{i\mu_l}{2(1 + \mu_l^2)} \int_0^\tau \left[\tilde{f}_l^n(w) - \tilde{f}_l^n(-w) \right] \cos(\beta_l(\tau - w)) dw.$$

In order to design an explicit scheme, we approximate the integral in (2.9) by the following quadrature,

$$(2.10) \quad \begin{aligned} &\int_0^\tau \left[\tilde{f}_l^n(w) - \tilde{f}_l^n(-w) \right] \cos(\beta_l(\tau - w)) dw \\ &\approx \frac{\tau}{2} \left[\tilde{f}_l^n(\tau) - \tilde{f}_l^n(-\tau) \right] \approx \frac{\tau}{2} \left[3\tilde{f}_l^n(0) + \tilde{f}_l^{n-2}(0) - 4\tilde{f}_l^{n-1}(0) \right]. \end{aligned}$$

Here, the first approximation is due to the trapezoidal rule and the second approximation is due to a backwards extrapolation. Note that when $n = 1$, we need an auxiliary approximation $u(x, t_{-1}) := u(x, -\tau)$, which can be achieved similar as (2.7) from (2.6) with $n = 0$ and $s = -\tau$,

$$(2.11) \quad \tilde{u}_l(t_{-1}) \approx \cos(\beta_l \tau) \widetilde{(u^{(0)})}_l - \frac{\sin(\beta_l \tau)}{\beta_l} \widetilde{(u^{(1)})}_l + \frac{i\mu_l \tau \sin(\beta_l \tau)}{2\beta_l(1 + \mu_l^2)} \widetilde{(u^{(0)}u^{(1)})}_l,$$

and subjecting (2.11) from (2.7), we have

$$\tilde{u}_l(t_{-1}) + \tilde{u}_l(t_1) = 2 \cos(\beta_l \tau) \widetilde{(u^{(0)})}_l.$$

2.2. Detailed numerical scheme. Let u_j^n be the approximation of $u(x_j, t_n)$ and denote by u^n the approximation vector with components u_j^n . Choosing $u_j^0 = u^{(0)}(x_j)$, the detailed numerical scheme computes u_j^n as

$$(2.12) \quad u_j^n = \sum_{l=-M/2}^{M/2-1} \widetilde{(u^n)}_l \exp\left(\frac{2ijl\pi}{M}\right), \quad j = 0, 1, \dots, M, \quad n = -1, 1, 2, \dots,$$

where,

$$(2.13) \quad \widetilde{(u^1)}_l = \cos(\beta_l \tau) \widetilde{(u^{(0)})}_l + \frac{\sin(\beta_l \tau)}{\beta_l} \widetilde{(u^{(1)})}_l - \frac{i\mu_l \tau \sin(\beta_l \tau)}{2\beta_l(1 + \mu_l^2)} \widetilde{(u^{(0)}u^{(1)})}_l,$$

$$(2.14) \quad \begin{aligned} \widetilde{(u^{-1})}_l &= -\widetilde{(u^1)}_l + 2 \cos(\beta_l \tau) \widetilde{(u^{(0)})}_l, \\ \widetilde{(u^{n+1})}_l &= -\widetilde{(u^{n-1})}_l + 2 \cos(\beta_l \tau) \widetilde{(u^{n+1})}_l \end{aligned}$$

$$(2.15) \quad -\frac{i\mu_l \tau}{4(1 + \mu_l^2)} \left[3\widetilde{(F^n)}_l + \widetilde{(F^{n-2})}_l - 4\widetilde{(F^{n-1})}_l \right], \quad n = 1, 2, \dots,$$

with $F^n = [(u_0^n)^2, (u_1^n)^2, \dots, (u_M^n)^2]^T$, and $\widetilde{(u^n)}_l$ the discrete Fourier transform coefficients for a vector $U = [U_0, U_1, \dots, U_M]^T$ satisfying $U_0 = U_M$, defined as

$$\tilde{U}_l = \frac{1}{M} \sum_{j=0}^{M-1} U_j \exp\left(-\frac{2ijl\pi}{M}\right), \quad l = -M/2, \dots, M/2 - 1.$$

It is readily to see this scheme is fully explicit and easy to implement, with memory cost $O(M)$ and computational load $O(M \ln M)$ per time step thanks to FFT.

Remark 2.1. *The scheme proposed in this section can be directly extended to the generalized symmetric regularized-long-wave equation [5],*

$$u_{tt} - u_{xx} + (g(u))_{xt} - u_{xxtt} = 0,$$

with $g(u)$ a general nonlinear function.

3. NUMERICAL RESULTS

In this section numerical results are reported towards testing the accuracy properties of the proposed numerical scheme (2.12)-(2.15), and applying it to study the collisions of solitary waves in SRLW. For all the results presented in this section, we always carry out the computation on an interval which is large enough such that the periodic boundary conditions do not introduce significant aliasing errors relative to the whole space problem.

TABLE 1. Spatial discretization error analysis: discrete l^2 -error in u at time $t = 10, 20$ under time step $\tau = 0.0001$.

time	$h = 2$	$h = 1$	$h = 1/2$
$t = 10$	1.5193E-1	1.9505E-4	1.4446E-6
$t = 20$	1.3929E-1	2.0105E-4	3.2758E-6

TABLE 2. Temporal discretization error analysis: discrete l^2 -error in u at time $t = 10, 20$ under mesh size $h = 1/8$.

time	$\tau = 0.04$	$\tau = 0.02$	$\tau = 0.01$	$\tau = 0.005$
$t = 10$	5.2720E-3	1.3140E-3	3.2810E-4	8.1985E-5
$t = 20$	8.4882E-3	2.0976E-3	5.2155E-4	1.3006E-4

TABLE 3. Conservation of invariants analysis: $I(u)$, $I(\rho)$, $E(u, \rho)$ and $V(u, \rho)$ at different time levels, with $h = 1/2$ and $\tau = 0.005$.

time	$I(u)$	$I(\rho)$	$E(u, \rho)$	$V(u, \rho)$
$t = 10$	12.0000038	8.4852811	27.1529477	16.8000335
$t = 20$	12.0000081	8.4852811	27.1529497	16.8000348
$t = 30$	12.0000124	8.4852811	27.1529515	16.8000361
$t = 40$	12.0000166	8.4852811	27.1529533	16.8000374

3.1. Accuracy tests. To test the accuracy properties, we take the initial conditions in (2.2) as the solitary waves (1.2) with $c = \sqrt{2}$, i.e.,

$$(3.1) \quad u^{(0)}(x) = \frac{3\sqrt{2}}{2} \operatorname{sech}^2\left(\frac{\sqrt{2}}{4}x\right), \quad u^{(1)}(x) = \frac{3\sqrt{2}}{2} \operatorname{sech}^2\left(\frac{\sqrt{2}}{4}x\right) \tanh\left(\frac{\sqrt{2}}{4}x\right).$$

To quantify the numerical results, we examine the standard discrete l^2 -error.

First, we test the discretization error in space, and in order to do this we take a very fine time step $\tau = 0.0001$ such that the error from temporal discretization is negligible compared to the spatial discretization errors. Tab. 1 lists the errors at time $t = 10, 20$ for different mesh size h . Next, we test the discretization error in time, and again we take a very fine mesh size $h = 1/8$ to eliminate the effect from spatial discretization. Tab. 2 shows the errors at time $t = 10, 20$ for different time step τ . Last, we test the conservative properties of the invariants given by (1.5)-(1.7). Tab. 3 lists the computed quantities (1.5)-(1.7) at different time levels, under mesh size $h = 1/2$ and time step $\tau = 0.005$. Note that for the example presented here, initial $\rho(x, 0)$ is given by $u^{(0)}/\sqrt{2}$ and during computation $\rho(x, t)$ is advanced by the standard midpoint rule with pseudospectral approximations of u_x . The integrals (1.5)-(1.7) are evaluated by the standard trapezoidal quadrature.

Based on these results, the following are drawn: (i). the scheme (2.12)-(2.15) is of spectral-order accuracy in space while second-order accuracy in time; (ii). the invariants defined by (1.5)-(1.7) are numerically conserved quite well over a long-time simulation.

3.2. Applications. Now some examples are reported to study the solitary wave collisions in SRLW by applying the proposed numerical scheme. We take the initial conditions in

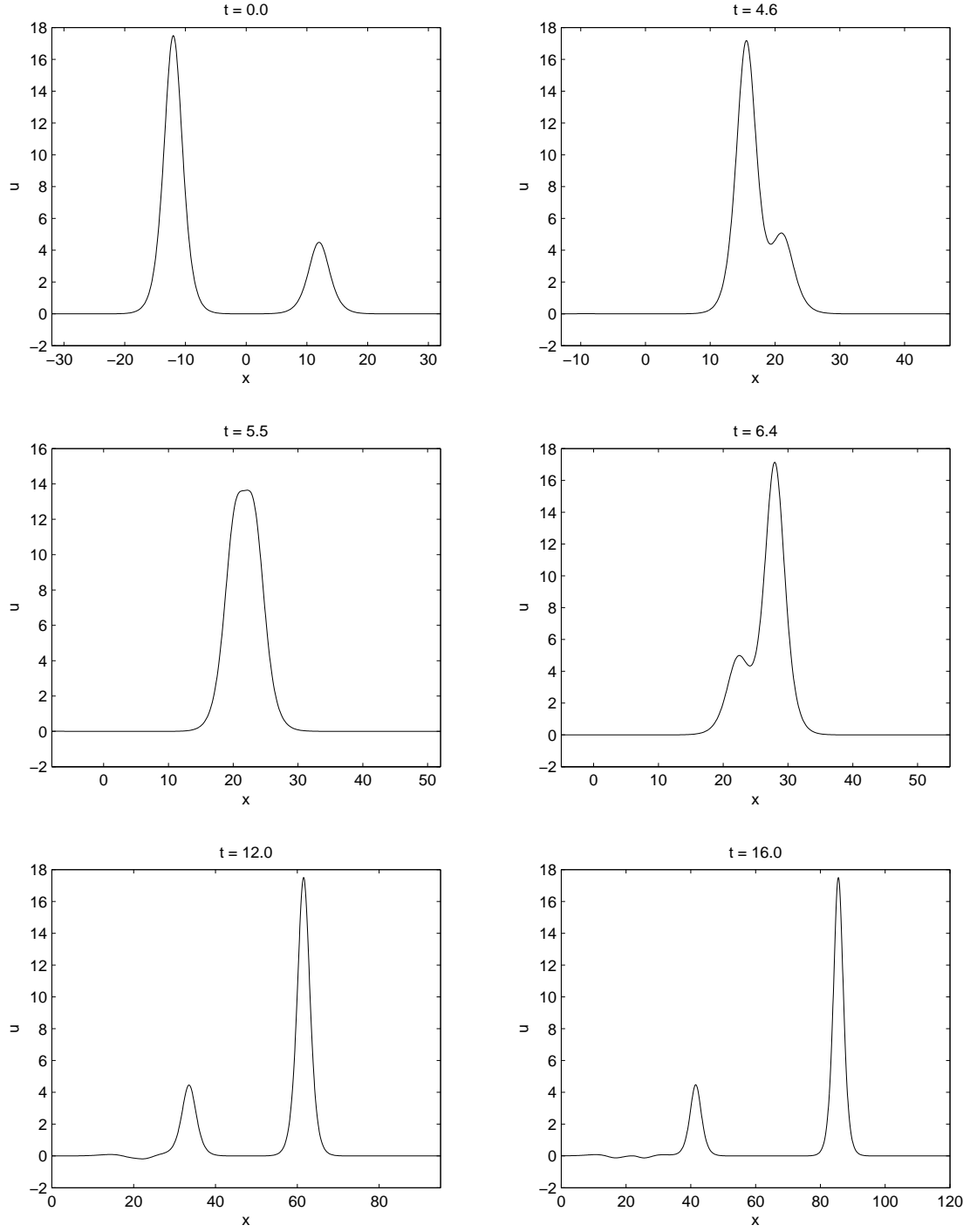


FIGURE 1. Overtaking collision of two solitary waves with velocity pair $(c_1 = 2, c_2 = 6)$ and $x_0 = 12$ in (3.2).

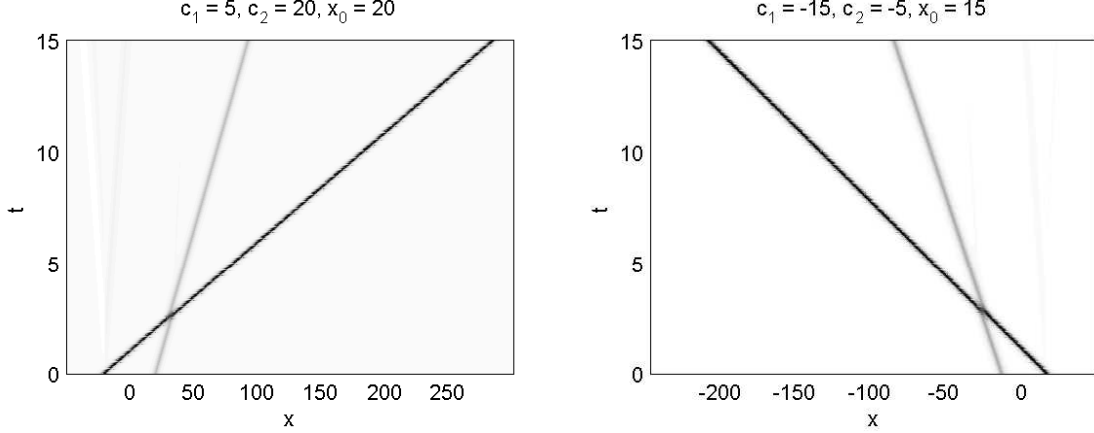


FIGURE 2. Contours of overtaking solitary wave collisions.

(2.2) as

$$(3.2) \quad u^{(0)}(x) = u_{c_1}(x - x_0, 0) + u_{c_2}(x + x_0, 0), \quad u^{(1)} = \partial_t u_{c_1}(x - x_0, 0) + \partial_t u_{c_2}(x + x_0, 0),$$

and consider various velocities pairs (c_1, c_2) to cover both cases of overtaking and head-on collisions.

- *CASE I: Overtaking solitary wave collisions.*

We consider the initial conditions (3.2) with velocities pair (c_1, c_2) such that $c_1 c_2 > 0$. Fig. 1 depicts the overtaking collision of two solitary waves with velocities $(c_1 = 2, c_2 = 6)$ and $x_0 = 12$ in (3.2). Fig. 2 shows the contours of the evolution of overtaking collisions for different initial solitary waves. Based on Figs. 1 and 2 and extensive examples, covering a wide range of velocities (c_1, c_2) , not shown here for brevity, overtaking solitary wave collisions are rather elastic with only a quite small amount of radiation emitted and no additional visible wave structures observed.

- *CASE II: Head-on solitary wave collisions.*

We consider the initial conditions (3.2) with velocities pair (c_1, c_2) such that $c_1 c_2 < 0$, and study two situations:

- (i). Initial two solitary waves with equal amplitudes, i.e., $c_1 = -c_2$. Fig. 3 depicts the head-on collision of two solitary waves with velocities $(c_1 = -15, c_2 = 15)$ and $x_0 = 20$ in (3.2). It shows that after the collision conspicuous radiations are emitted and a number of soliton pairs are produced from either side of the radiation. For instance, at time $t = 11.8$ four soliton pairs are readily detectable and at time $t = 30.0$ five conspicuous soliton pairs are observed. Similar results are also observed for other choices of initial equal amplitudes, some of which are plotted out in Fig. 4. It suggests that there is no upper limit on the number of emerging soliton pairs, which indeed appears to be a non-decreasing function in the argument of velocity amplitude $|c_1|$ (or $|c_2|$).
- (ii). Initial two solitary waves with unequal amplitudes, i.e., $c_1 \neq -c_2$. Fig. 5 shows the contour and evolution of the head-on collision of two solitary waves with velocities $(c_1 = -20, c_2 = 18)$ and $x_0 = 20$ in (3.2). It shows that after the collision conspicuous radiations are emitted with an additional soliton pair emerging from either side of the radiation. It is interesting to point out that in

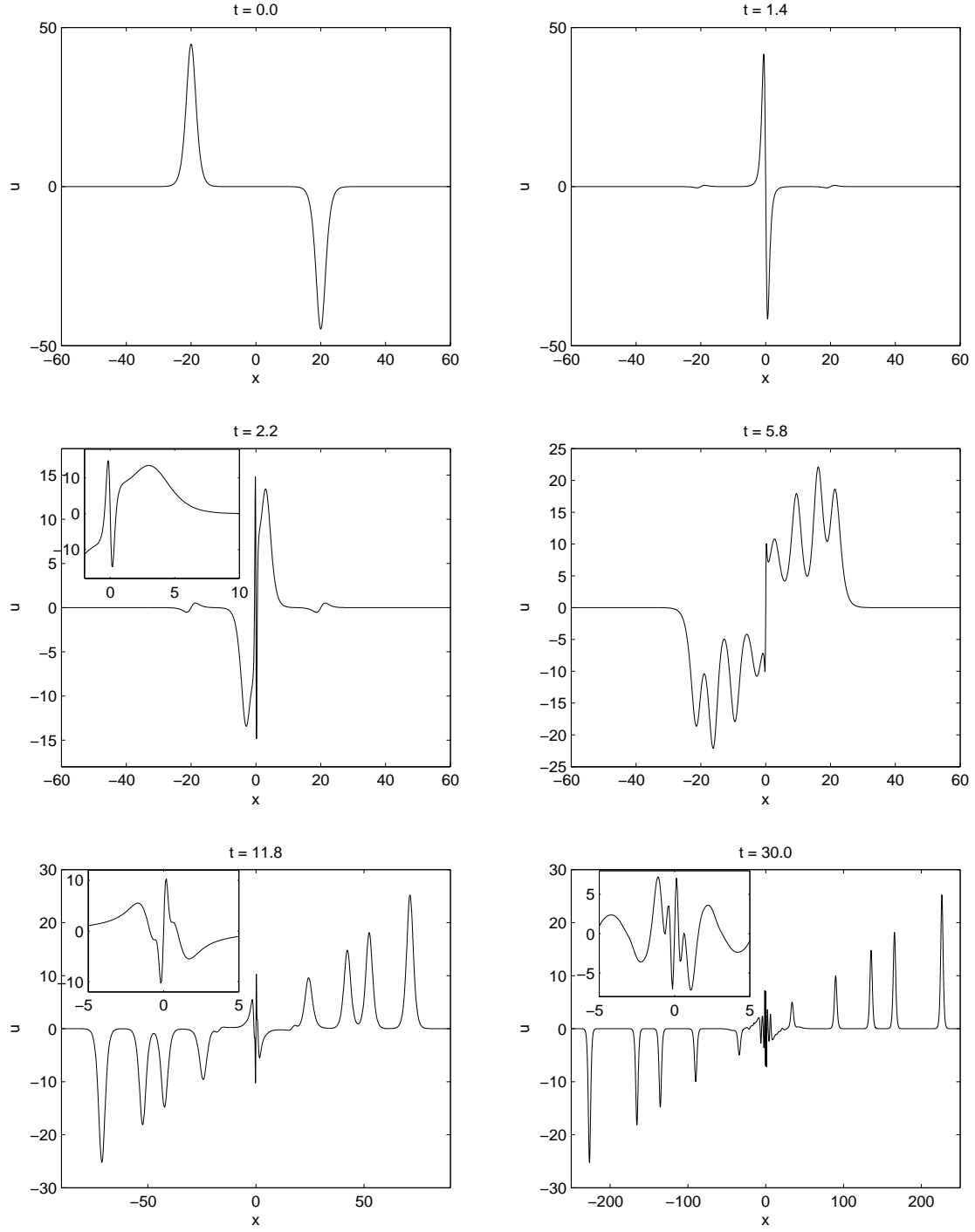


FIGURE 3. Head-on collision of two equal-amplitude solitary waves with velocity pair $(c_1 = -15, c_2 = 15)$ and $x_0 = 20$ in (3.2).

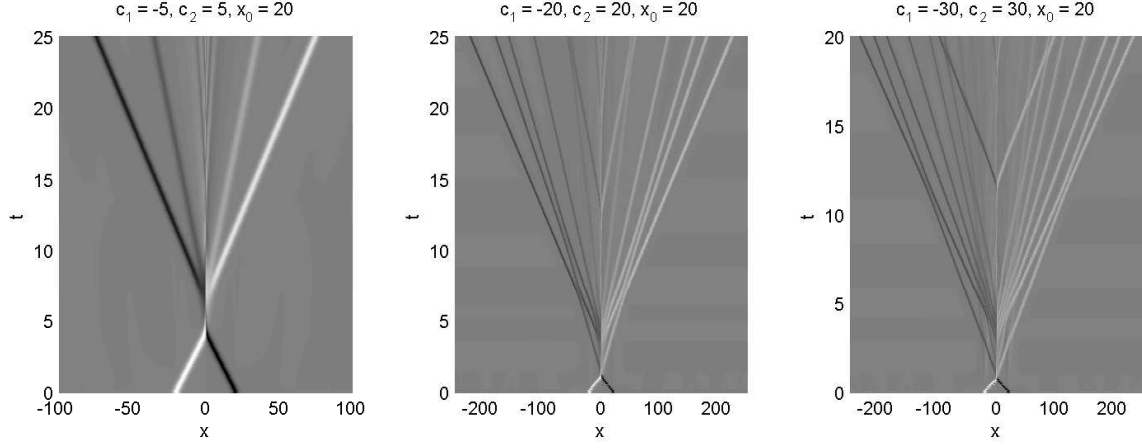


FIGURE 4. Contours of head-on equal-amplitude solitary wave collisions.

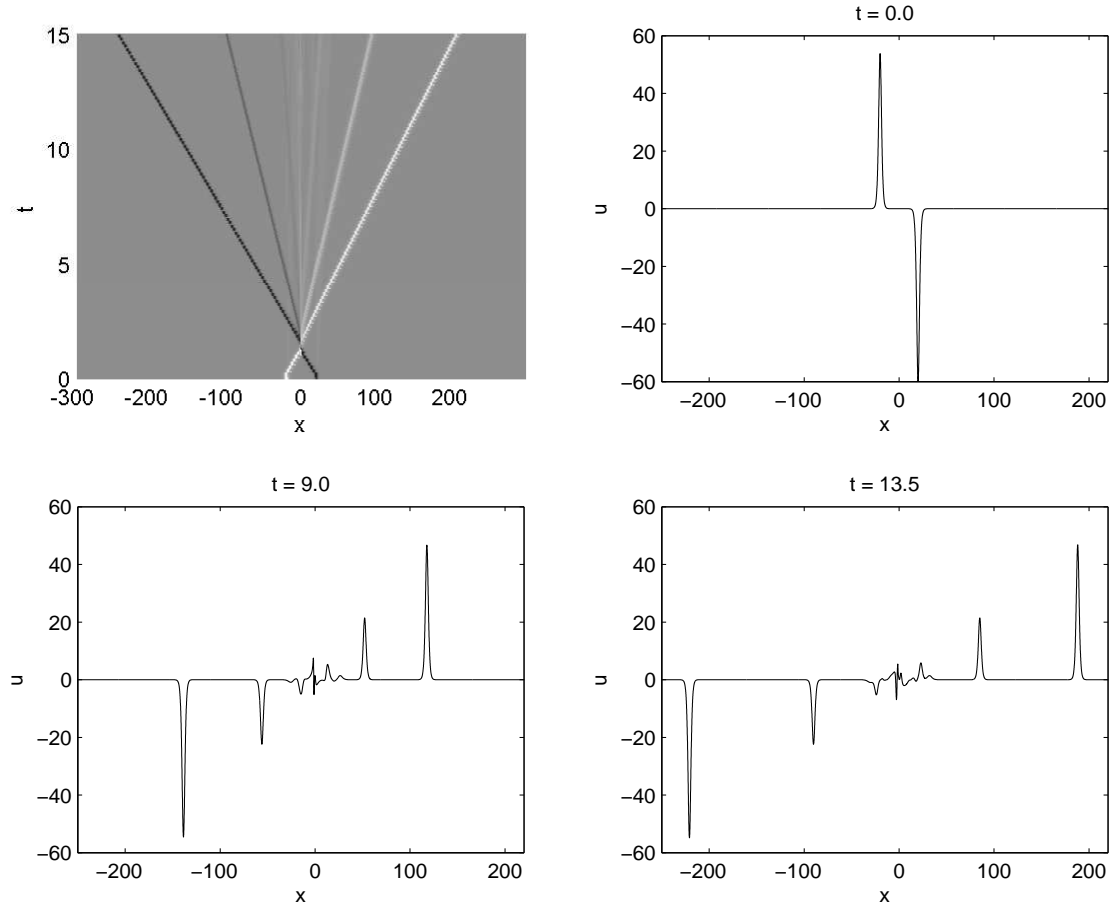


FIGURE 5. Head-on collision of two unequal-amplitude solitary waves with velocity pair $(c_1 = -20, c_2 = 18)$ and $x_0 = 20$ in (3.2).

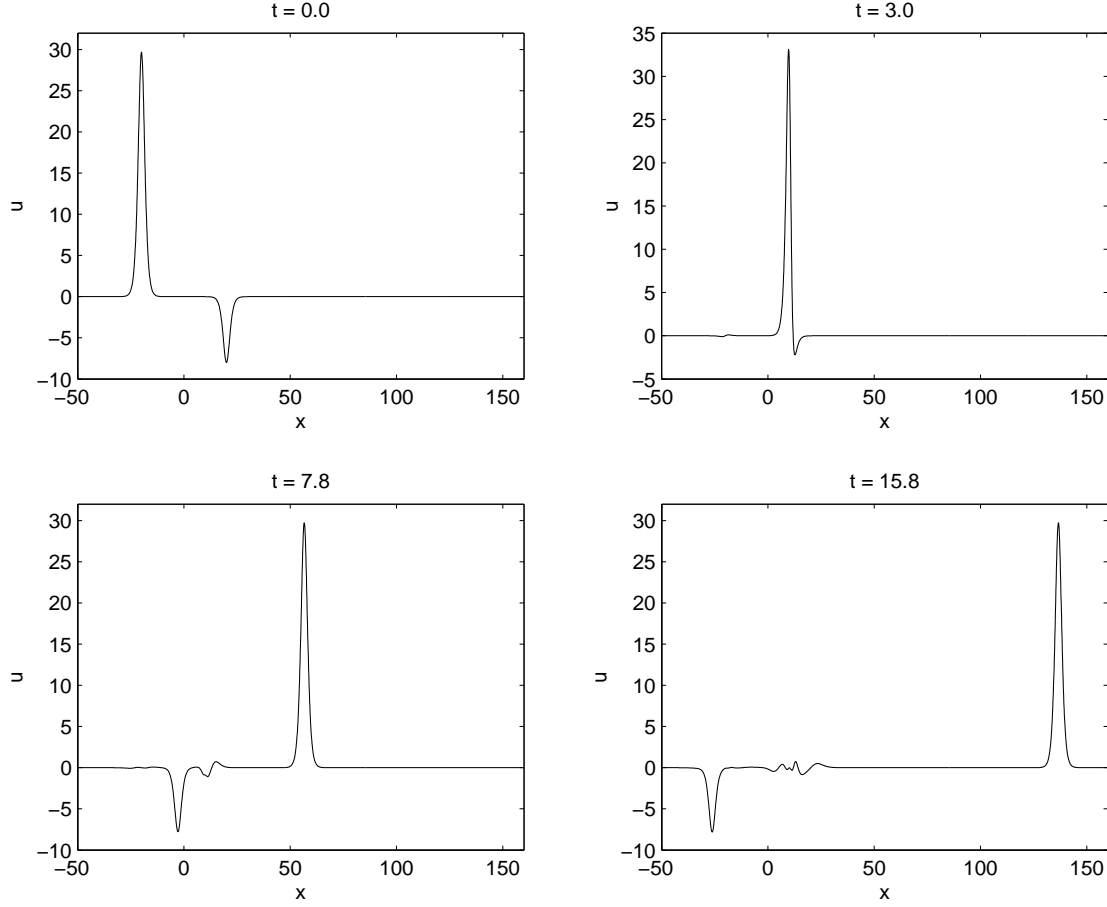


FIGURE 6. Head-on collision of two unequal-amplitude solitary waves with velocity pair $(c_1 = -3, c_2 = 10)$ and $x_0 = 20$ in (3.2).

this example although the initial amplitudes are nearly equal, the propagation result differs much from that of equal-amplitude solitary waves. The head-on collision of equal-amplitude solitary waves with $(c_1 = -20, c_2 = 20)$ produces four additional soliton pairs after the collision (see Fig. 4), while in Fig. 5 only one additional soliton pair emerges. Fig. 6 shows the case $(c_1 = -3, c_2 = 10)$, in which the collision is nearly elastic with only a small amount of radiation emitted.

The results presented in this section suggest that the collisions of solitary waves in the SRLW equation are rather complicated issues. Also, these results well demonstrate the efficiency and high resolution of the proposed scheme (2.12)-(2.15) for the numerical study of the SRLW equation.

4. CONCLUSIONS

We solved the symmetric regularized-long-wave (SRLW) equation with the utility of pseudospectral discretization in space combined with a time integration based on trigonometric polynomials. The proposed numerical scheme is of second-order accuracy in time and

spectral-order accuracy in space. The collisions of solitary waves in the SRLW were extensively investigated via numerical simulations. The method and results discussed in this work are applicable to the physical problem of nonlinear space-charge wave propagation on electron beams.

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